

# Supply, Demand and Market Impact

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## What is Price Impact?

- Price impact = correlation between an arriving order and the subsequent price change
- Sadly enough: *on average* buy (sell) trades push the price up (down)
- THIS IS HIGHLY RELEVANT:
  - induces extra **execution costs** (large but often overlooked)
  - limits the **capacity** of strategies (costs increase with size)
  - makes marked-to-market valuation **over-optimistic**
  - can lead to crashes (the impact of a trade can trigger other trades)

**Intuition: price impact is inversely related to market liquidity**

**SURE, BUT HOW EXACTLY ?**

▪ Liquidity? What liquidity?

- Immediate liquidity at any given moment is small, and affected by tick size, priority rules, fees, market makers, HFT, etc.
    - ✓ For a liquid stock the instantaneous volume in the order book is approx.  $10^{-6}$  of market cap., when the total daily traded volume is 5000 times larger (x5 since 1960)
  - Most of the available volume is “latent”, and only progressively gets revealed during the day
  - Large trades must be sliced and diced and executed incrementally
- **What is the (average) impact of a metaorder of size  $Q$ ?**

# IMPACT OF METAORDERS

## A Universal Empirical Result ?

A metaorder of size  $Q$  has a price impact:

$$I(Q) = Y \sigma_T \sqrt{\frac{Q}{V_T}}$$

where:

$Q$  is the volume of the metaorder

$\sigma_T$  is the volatility of the market

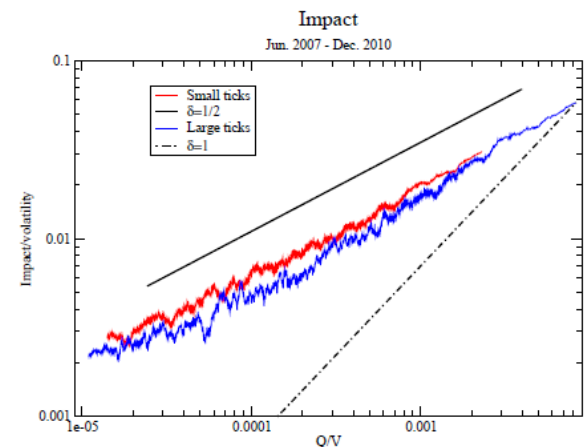
$V_T$  is the total volume traded in the market

( $Y$  of order 1)

Consistently reported by many groups since the mid-nineties:

BARRA, Almgren, Engle, JPM, DB, LH, CFM – different strategies, different markets, different execution protocols (limit vs market orders, etc.), different tick sizes, different analysis of data.

Very stable  $Y$  ratio across time (CFM)



# SQRT(Q): A VERY STRANGE IMPACT LAW

- Impact is concave (not additive):

→

$$1+1 = 1.4142 < 2$$

The impact of the last  $Q/2$  is 60% of the first  $Q/2$  !

→ **Anomalously large impact of small trades:** 1% of ADV pushes the price by 10% of its vol

- Important note: impact is usually small compared to vol

→ often goes unnoticed, but a systematic detrimental effect!

## MORE ON THE SQRT(Q) IMPACT LAW

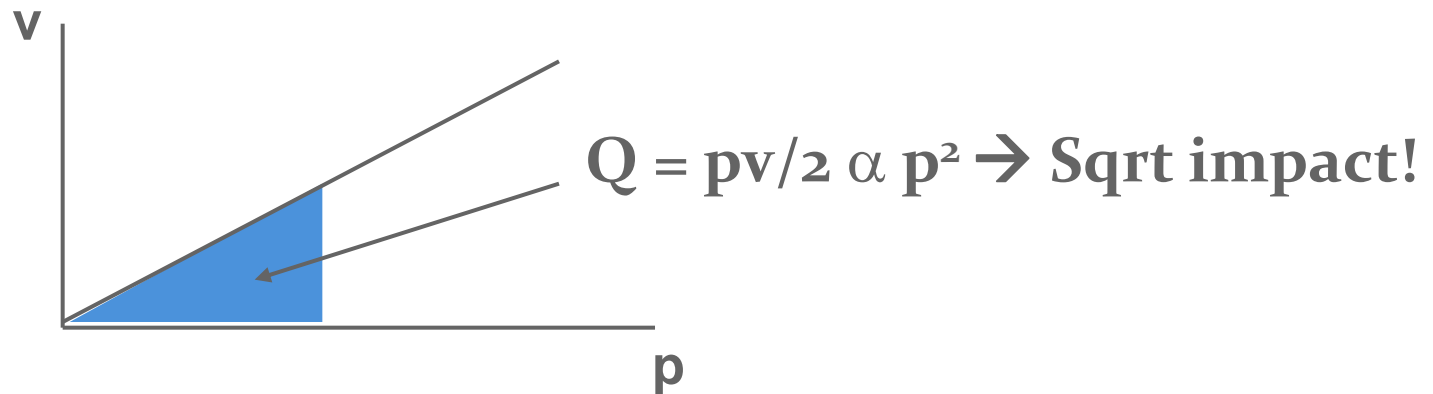
$$I(Q) = Y \sigma_T \sqrt{\frac{Q}{V_T}}$$

- **Remarkable stability of results:** style of trading, strategies, markets, period (1995 → 2012), tick sizes, treatment of data, etc. – hints that microstructure and HFT effects are not relevant, only “macro-liquidity”
- Impact is, to a first approximation, independent on the time to complete the metaorder (!), only on Q
- **A genuine “physical law” of financial markets? Why?**

# LINEAR SUPPLY/DEMAND

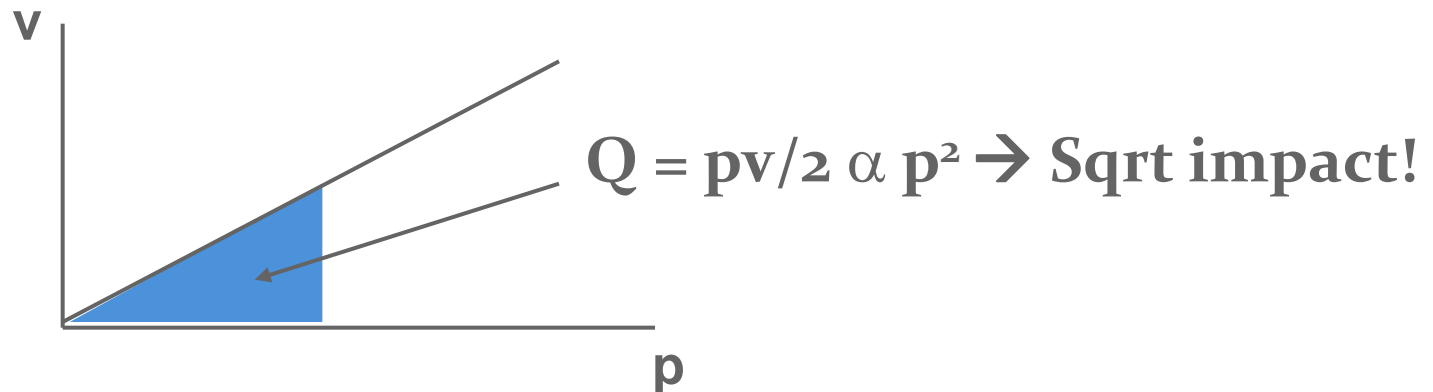
## Intuition:

- Impact must be limited by the volume on the other side
- Assume *by fiat* volume of opposite sellers is linear in price
- More resistance (less impact) as the price increases



# A DYNAMICAL THEORY OF (LATENT) LIQUIDITY

- But why should the supply/demand profile be *linear* and vanish around the *current* price ?
- Many theories since the late 90's about rational agents/fundamental price/optimizing market makers/...all very ad hoc...
- Our theory\*: a purely **statistical** effect, even with “zero-intelligence” trades: provided the price makes a *random walk*, and for a *generic order flow*, the probability to have an unexecuted (latent) order close to the current price is indeed **linearly** small !!



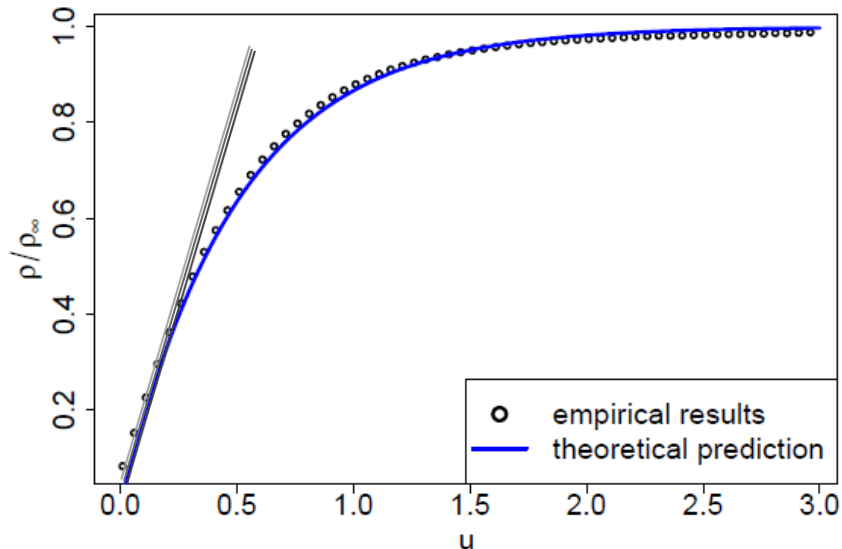
\* B. Toth, et al. PRX 1, 021006 (2011)



# A NUMERICAL “AGENT BASED” MODEL

## An “Agent Based” Numerical Model to Test the Theory

- People decide randomly on orders to buy or to sell and their price level
  - These orders are “eaten” by transactions
  - Realistic statistics for order flow (correlations, opportunistic)
  - No fundamental prices, no fancy behavioral assumptions
- only random walk prices



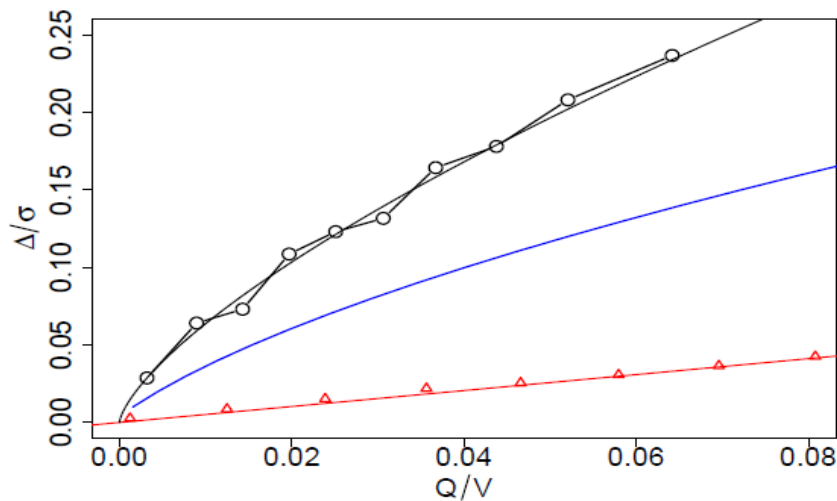
# RESULTS OF THE MODEL

Let's now add an Extra Buyer in this Artificial Market

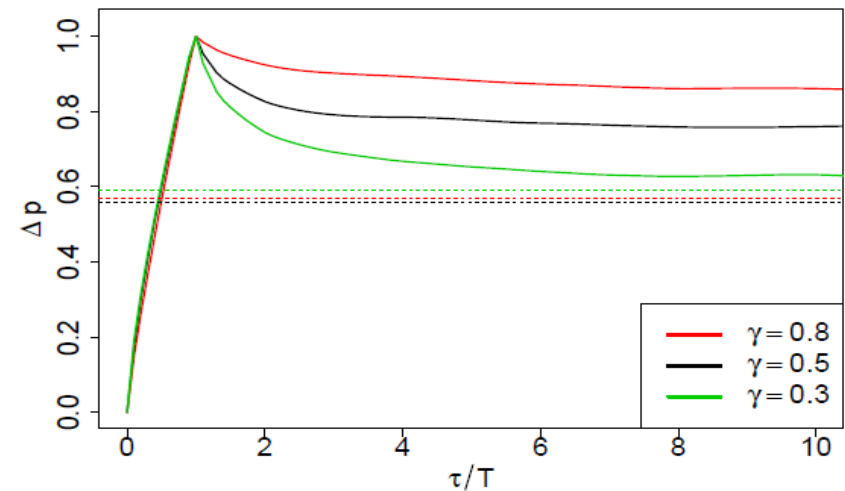
Results

$$I(Q) = Y \sigma_T \sqrt{\frac{Q}{V_T}}$$

1) Square-Root Impact



2) Decay of Impact



# SO WHAT? I: THE TRUE COST OF TRADING

$$I(Q) = Y \sigma_T \sqrt{\frac{Q}{V_T}}$$

- Naïve answer: the bid-ask spread (sensitive to microstructure, etc.)
- True for small trades, but as size grows, impact costs quickly dominate (although often disregarded)
- Order of magnitude: for  $Q=1\%$  of daily volume and 2% vol:

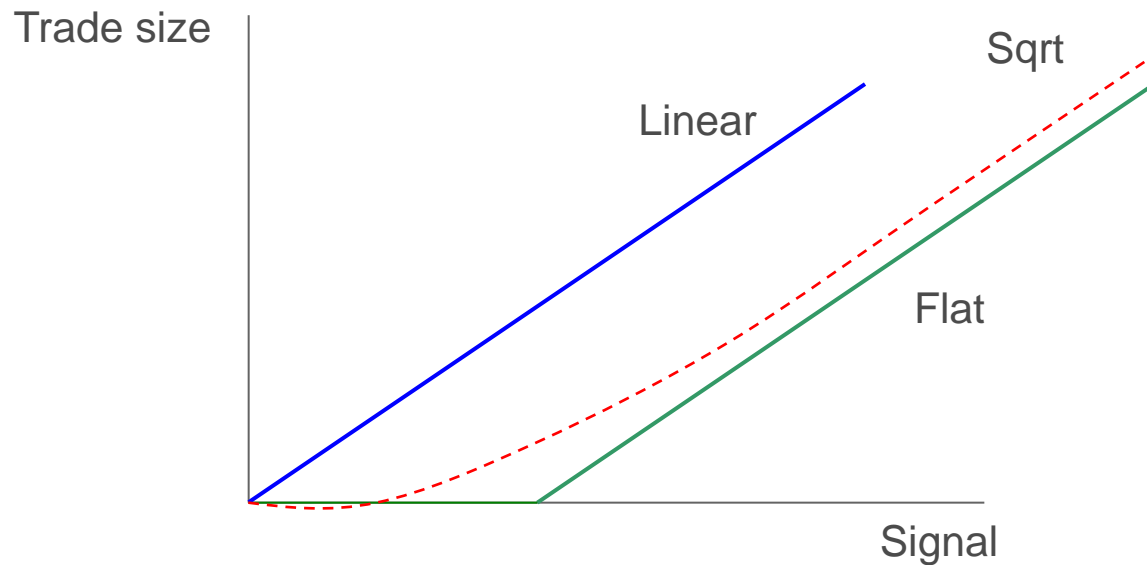
$$\text{Cost} = 1 \text{ bp} + 0.5 * 2\% * \text{sqrt}(1\%) = 1 + 14 \text{ bp}$$

- Impact is dominated by “true liquidity” and very little by microstructure
- Many complaints about HFT have **misplaced focus**: impact is **unavoidable** and much larger than spreads! Dark pools are no solution!

## SO WHAT? II: TRADE SELECTION

$$I(Q) = Y\sigma_T \sqrt{\frac{Q}{V_T}}$$

- Affects the size of the optimal trade

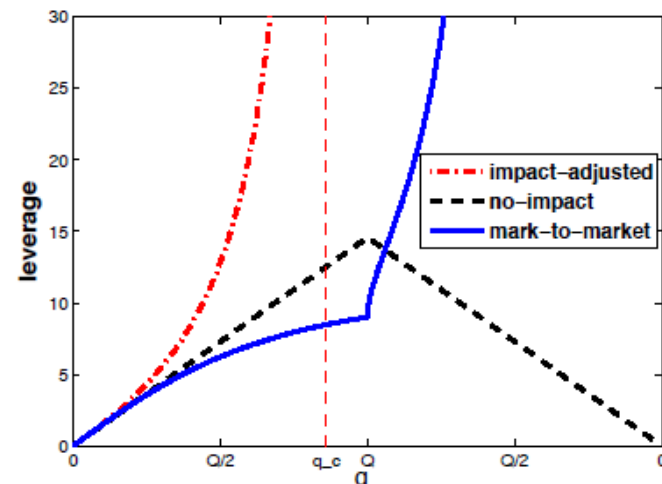
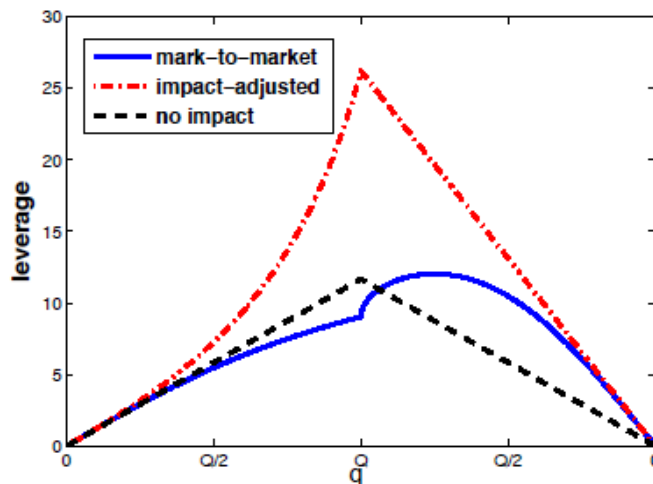


➤ **Optimum: Costs = 2/3 of expected gains !!!**

## SO WHAT? III: DELEVERAGING TRAP

$$I(Q) = Y\sigma_T \sqrt{\frac{Q}{V_T}}$$

- When selling in an attempt to deleverage, impact drives down the value of the remaining assets and can in fact increase leverage!
- An **impact-adjusted mark-to-market** accounting rule may avoid bad surprises



# SO WHAT? IV: INTRINSIC MARKET FRAGILITY

## Broader Consequences for Market stability/fragility

- Liquidity at the best price is vanishingly small (it is “**eaten up**” by the diffusive motion of prices)
- This imposes a **splitting and dicing of metaorders...**
- ...and leads to an **anomalously large impact for small trades**
- Liquidity fluctuations are bound to play a crucial role:  
→ **Micro-crises and jumps in prices** without news, as indeed seen empirically – a large fraction of volatility appears to be self-referential

(cf Hawkes process)

